The hypoelliptic Laplacian of J.-M. Bismut

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In recent works, Jean-Michel. Bismut ([1],[2],[3]) has introduced a new operator (the hypoelliptic Laplacian) on the cotangent bundle T^*X of a Riemannian variety X which is a deformation of the Hodge Laplacian on X. This operator is a particular case of Geometric Fokker-Planck equations (GFK) on T^*X. These equations are the kinetic version on T^*X of Laplacian operators defined on X. They can be view as generalized Kolmogorov equations, and are hypoelliptic operators on T^*X . There exist many papers devoted to the study of Fokker-Planck equations acting on functions (0-forms) in the case of flat metric on the Euclidian space $\mathbb{R}^n$ and with a potential, both from the PDE and probabilistic point of view. (see [5], [6] for a recent PDE study).

In this talk, we will give an introduction to GFK operators acting on differential forms on the cotangent bundle T^*X of a Riemannian variety X, and we will explain the strategy of proof of the analytic convergence of the hypoelliptic Laplacian toward the Hodge Laplacian which is part of the work [4].

Références

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