Inverse image of D-modules and weighted b-functions

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Weighted b-functions where introduced in a joined work with E.Galina (Duke Math. J., 123, 2 (2004) p 265-309). They are analogous to the usual b-functions but the Euler vector field $\sum x_i D_{x_i}$ is replaced by a vector field like $\sum m_i x_i D_{x_i}$ for strictly positive integers m_i. We show here that we can calculate the weighted b-functions of the inverse image of a holonomic D-module in cases where this is not possible with usual b-functions. The first example is the case of a ramification map $(x_1, \ldots, x_n) \mapsto (x_1^{m_1}, \ldots, x_n^{m_n})$.

If $\mathfrak{g}$ is a semi-simple Lie algebra, Hotta and Kashiwara defined a holonomic D-module $\mathcal{M}_\lambda$ whose solutions are the invariant eigendistributions on $\mathfrak{g}$, this definition was extended to symmetric pairs by Sekiguchi. We apply the previous result to the inverse image of $\mathcal{M}_\lambda$ to the Springer resolution of $\mathfrak{g}$ (or its extension to symmetric pairs) and improve our results of loc.cit. on the integrability of the solutions of $\mathcal{M}_\lambda$. In fact, our result was optimal in the case of a semi-simple Lie algebra but not in the case of symmetric pairs.