Thm (Lin-Phillips 2004)

X : compact, metrizable finite dim.

$\alpha \in \mathrm{Homeo}(X)$: minimal

$A := C^*(X, \alpha)$

T.F.A.E.

(1) $\rho(K_0(A)) \subset \mathrm{Aff}(T(A))$ is dense

(2) A has real rank zero

(3) A is tracially AF

X : the Cantor set

$\cong \{0, 1\}^{\mathbb{Z}}$

$\mathbb{T} := \mathbb{R}/\mathbb{Z}$

$\gamma \in \mathrm{Homeo}(X \times \mathbb{T})$

$\rightsquigarrow {}^{\exists}\alpha \in \mathrm{Homeo}(X)$

${}^{\exists}\varphi : X \to \mathrm{Homeo}(\mathbb{T})$

s.t. $\gamma(x, t) = (\alpha(x), \varphi_x(t))$

γ : minimal $\Rightarrow$ (X, α)

Cantor minimal system

$$F : X \times \mathbb{T} \to X$$

$$F \circ (\alpha \times \varphi) = \alpha \circ F$$

factor map

Def.

$\alpha \times \varphi$: rigid

$\Longleftrightarrow$ $F_* : M_{\alpha \times \varphi} \to M_\alpha$

is injective

Thm (Lin - M)

$\alpha \times \varphi$: minimal

$A := C^*(X \times \mathbb{T}, \alpha \times \varphi)$

$\alpha \times \varphi$: rigid

$\Longleftrightarrow$ A has real rank zero

$$R_s : \mathbb{T} \longrightarrow \mathbb{T}$$
$$t \mapsto t+s$$ Translation

$\xi : X \to \mathbb{T}$; conti.

$$\alpha \times R_\xi (x, t) := (\alpha(x), t + \xi(x))$$

Thm

$\xi : X \to \mathbb{T}$, conti.

$\alpha \times R_\xi$; minimal

$A := C^*(X \times \mathbb{T}, \alpha \times R_\xi)$

T.F.A.E.

(1) $\alpha \times R_\xi$: rigid

(2) A has real rank zero

(3) A is tracially AF

$\varphi : X \to \mathrm{Homeo}(\mathbb{T})$

$$o(\varphi)(x) := \begin{cases} 0 & \varphi_x \in \mathrm{Homeo}^+(\mathbb{T}) \\ 1 & \text{otherwise} \end{cases}$$

$\rightsquigarrow [o(\varphi)] \in K^0(X, \alpha) \otimes \mathbb{Z}_2$

Def.

$\alpha \times \varphi$ is orientation preserving

$\iff [o(\varphi)] = 0$

Lem

$\alpha \times \varphi$: minimal, not o.p.

$\Longrightarrow \alpha \times o(\varphi) \times \varphi \in \mathrm{Homeo}(X \times \mathbb{Z}_2 \times \mathbb{T})$

$(x, k, t) \mapsto (\alpha(x), k + o(\varphi)(x), \varphi_x(t))$

is minimal, o.p.

$A := C^*(X \times \mathbb{T}, \alpha \times \varphi)$

$B := C^*(X \times \mathbb{Z}_2 \times \mathbb{T}, \alpha \times o(\varphi) \times \varphi)$

$$(x, k, t) \mapsto (x, k+1, t)$$

induces $\theta \in \mathrm{Aut}(B)$

and $B \rtimes_\theta \mathbb{Z}_2 \cong M_2(A)$

Thm

$\theta \in \mathrm{Aut}(B)$ has

tracial Rohlin property

Cor

$\varphi : X \to \mathrm{Isom}(\mathbb{T})$

$\alpha \times \varphi$: minimal

この場合でも

real rank zero

$\Longleftrightarrow$ tracially AF

$\Longleftrightarrow$ rigid